Empirical Analysis of Stock Returns

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Lecture Overview

1. The Similarities and Differences between CAPM and APT
2. Test of the CAPM
3. Test of the APT
4. Fama-French Three-Factor Model
5. Carhart Four-Factor Model
6. Fama-French Five-Factor Model
7. Hou, Xue, and Zhang's q-Factor Model

CAPM vs. APT

- Similarities:
  - Both serve as investment benchmarks
  - Both split risk into non-diversifiable component rewarded with premium and diversifiable component that is not rewarded.
- Differences:
  - Market equilibrium under CAPM is maintained through mean-variance efficiency, under APT through arbitrage.
  - CAPM requires unobservable market portfolio while APT gives no theoretical guidance concerning the nature of factors.
  - These differences are not very relevant for empirical tests, since the CAPM is most often tested through single-index models.
  - The reason is that the market portfolio is not observed, so it has to be “proxied” by a market index.
  - Therefore, empirical test tend to simultaneously access the validity of the CAPM and of the APT.

Test of the CAPM

- Using the Security Market Line (SML)
- Hypothesis : average returns (in a cross section of stocks) depend linearly and solely on asset betas.

- Expected Return-Beta Relationship

$$E[r_i] - r_f = \beta_i (E[r_M] - r_f)$$

- Estimating the Security Characteristic Line

$$r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{Mt} - r_{ft} \right) + e_{it}$$

Test of the CAPM:

- Expected Return-Beta Relationship

$$E[r_i] - r_f = \beta_i (E[r_M] - r_f)$$

- Two-Stage Fama-MacBeth Cross-Sectional Regressions

- Step 1: Establishing sample data
  - Sample period:1927-2019
  - Cross-section: returns of NYSE, Amex and Nasdaq stocks
  - Market portfolio proxy: VW CRSP market index
  - Risk free asset: 3-month Treasury Bill

Test of the CAPM:

- Step 2: The First-Stage Time-Series Regression:
  - Estimate beta through time-series regression

$$r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{Mt} - r_{ft} \right) + e_{it}$$

- where r_{it}, r_{ft} and r_{mt} are the returns of stock *i*, risk free asset, and the market index respectively at month *t*; α_i is abnormal/risk-adjusted returns (regression intercept); β_i is systematic risk/nondiversifiable risk (regression slope coefficient); e_{it} is nonsystematic risk/firm specific risk/idiosyncratic risk (regression residual).
- Using the historical return data over the chosen sample period, we obtain a beta estimate for each stock in the cross-section of the sample.

Test of the CAPM:

- Step 3: The Second-Stage Cross-Sectional Regression
  - Use estimates from the first-stage time-series regression to estimate risk premium through cross-sectional regression.
  - In the other words, we use the cross-section of stocks and their average excess return, beta (from the first-stage time-series regression) to estimate the expected return-beta relationship or the slope of the SML.
  - Coefficients indicate the relationship.

Overview of Investigation

Two problems of empirically testing CAPM

1) Market index proxy (Roll's Critics)
   - The only testable hypothesis is whether the market portfolio is mean-variance efficient.
   - If this assumption does not hold, other predictions of CAPM are not testable separately.
   - CAPM is not testable unless we know the exact composition of the market portfolio and use it in the tests.
   - Empirical proxies of market portfolio such as S&P 500 may not be mean-variance efficient while market portfolio is (benchmark error).

1) 23

2) Beta estimation error

- Statistical problems caused by measurement error in the estimation of beta in the first-stage time-series regression.

Solution: to reduce beta measurement error

- Portfolio diversify away company-specific noise, thus leading to more precise beta estimates.
- Therefore estimate betas in the first-stage regression for portfolios instead of individual stocks.
- The problem with this approach is that it leaves the second-stage cross-sectional regression with narrower cross-section.
- Widen cross-sectional variations of beta in the second-stage regression by forming portfolios first sorted by betas.

Test of the APT

- A multifactor capital market usually is postulated.

- The multifactor APT model test:
  - Problem of APT is the lack of guidance concerning which factors ought to result in risk premiums.
  - Three stages of test

    Step 1: Specification of risk factors

    Step 2: Identification of portfolio that hedge those fundamental risk factors.

    Step 3: Test of the explanatory power and risk premium of the hedge portfolios.

Motivation of Fama-French Three-Factor Model

- Banz (1981): **Firm size** can explain the cross-section of average returns in addition to Beta.
- Bhandari (1988): **leverage** helps explain the cross-section of average stock returns in tests that include size and Beta.
- Rosenberg, Reid, and Lanstein (1985): **book-to-market** equity (BE/ME) also has a strong role in explaining the cross-section of average returns.
- Basu (1983) shows that **earnings-price ratios** (E/P) help explain the cross-section of average returns.

The Size Effect

- Banz (1981, Journal of Financial Economics) examines the relationship between the return and the total market value of NYSE common stocks.
- He finds that on average, small-capitalization firms earned higher returns than large-capitalization firms.
- He shows that small-capitalization firms on the NYSE earned higher returns than is predicted by the CAPM.

The Size Effect

- Group all stocks each year into 10 portfolios, sorted on their market capitalization(=ME deciles)

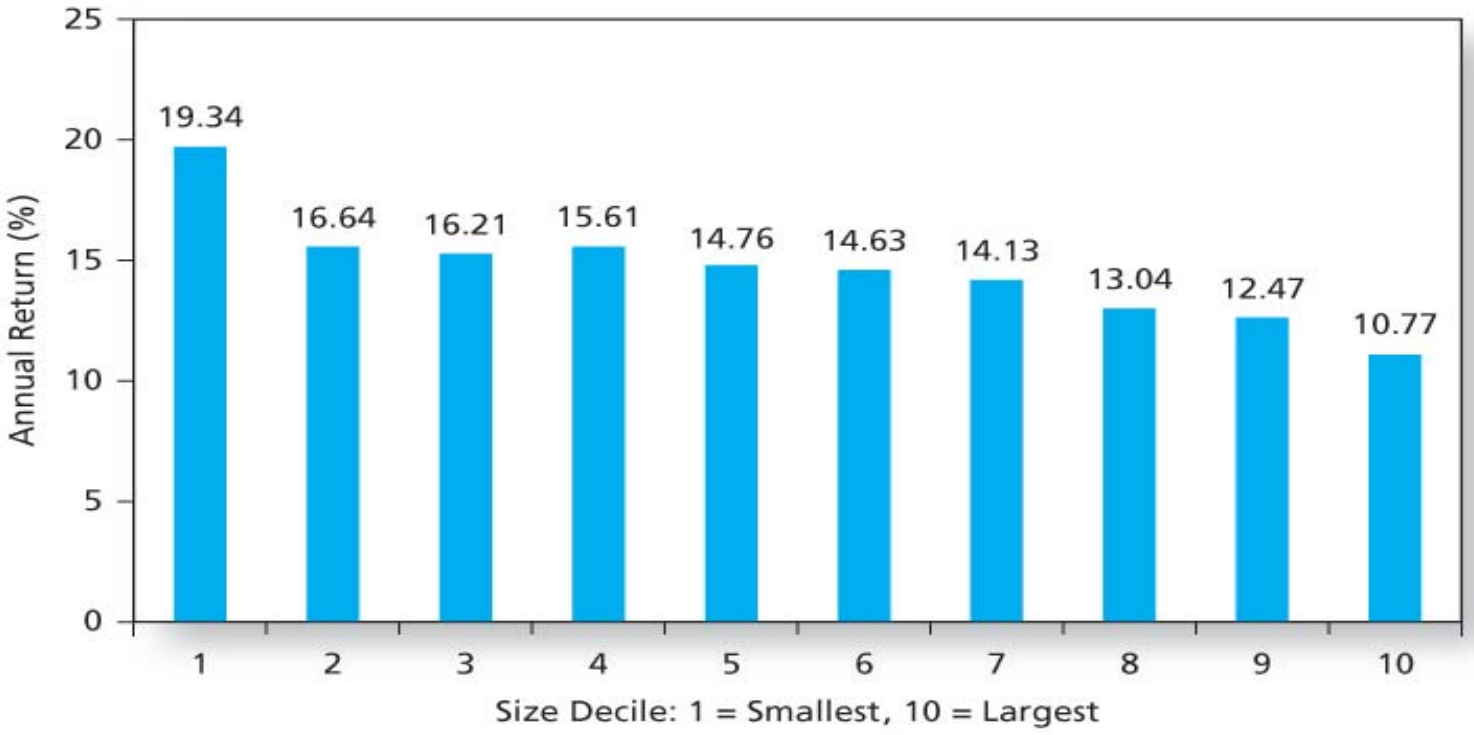
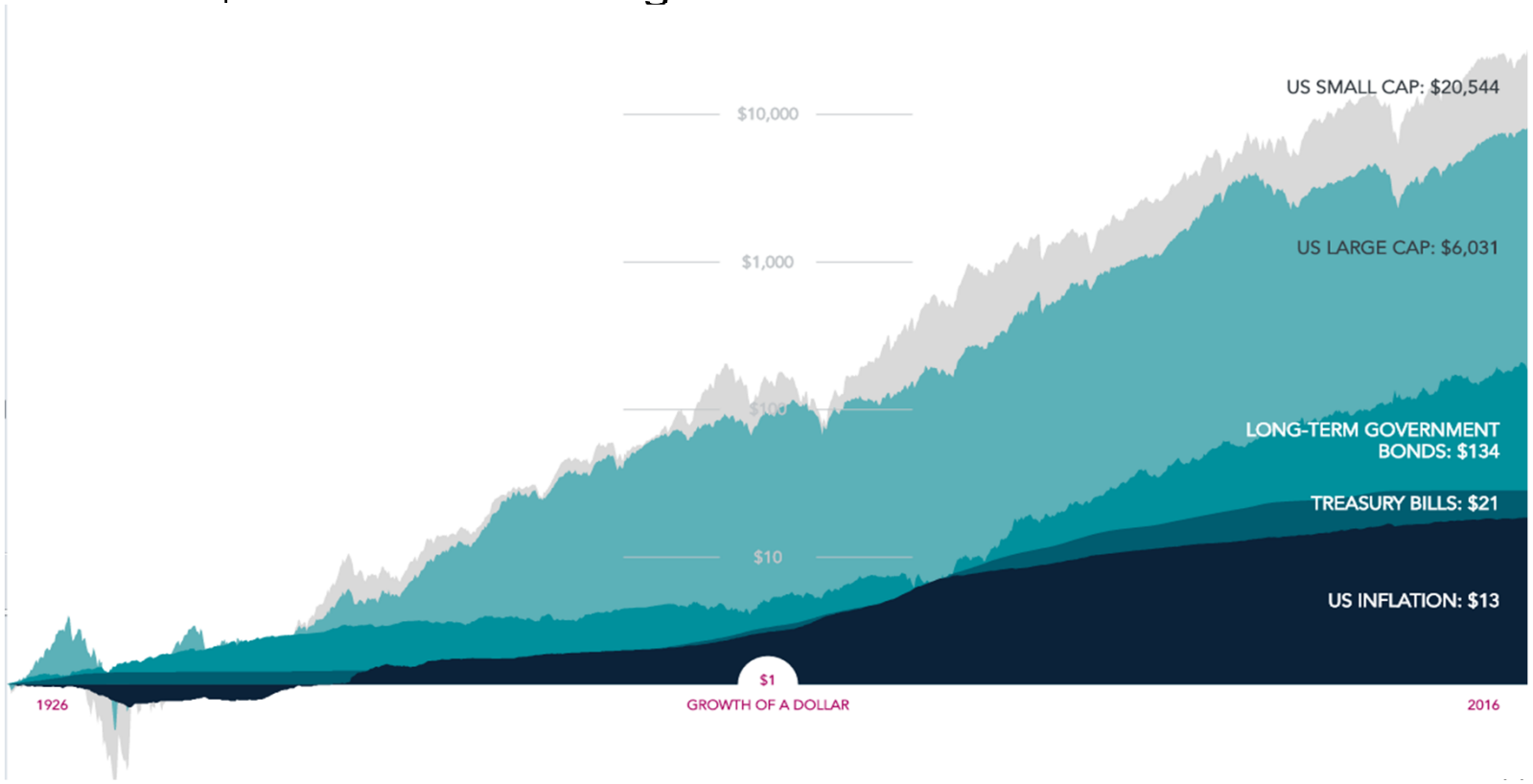


Figure 11.3 Average annual return for 10 size-based portfolios, 1926–2008

Source: Authors' calculations, using data obtained from Professor Ken French's data library at **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

The Size Effect

- The financial community picked up on the size effect, since Dimensional Fund Advisors (DFA) began in 1981 with Eugene Fama as their Director of Research.
- Growth of $1 from 1926 through 2016:



Source: Dimensional Fund Advisors

The Value Effect

- **Value Stocks:**
  - Stocks with high book-to-market ratio: Book-value-of-firm / market-value-of-firm
- **Growth Stocks:**
  - Stocks with low book-to-market ratio.
  - Stocks with large growth in earnings and revenue. Typically pay small dividends – they use it to finance expansion instead.
- Why buy value stocks?
  - Because they are “cheap”
- Why buy growth stocks?
  - Because of their potential earnings.

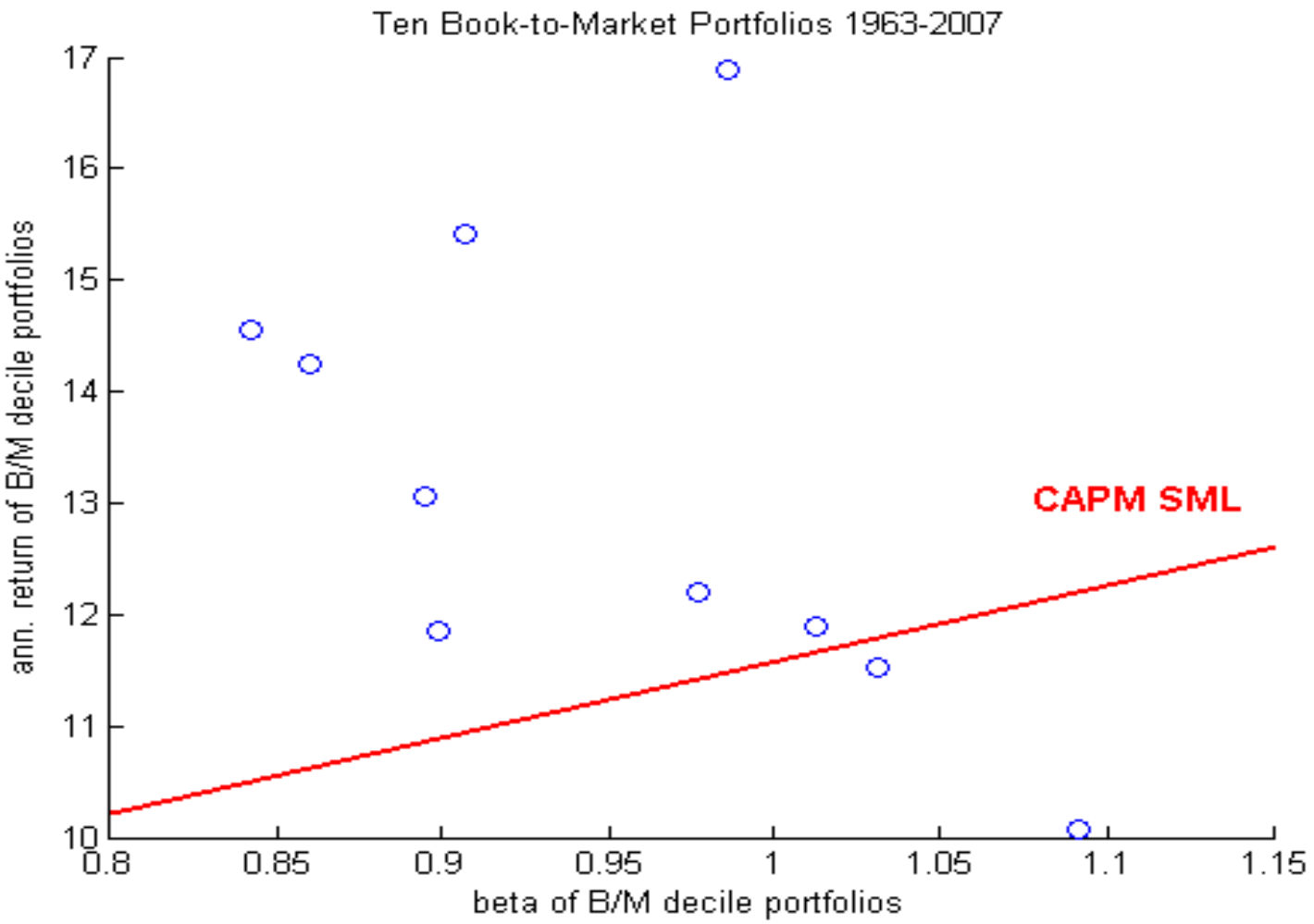
The Value Effect in detail

- Group all stocks each year in 10 portfolios, sorted on their Book-to-Market ratio (=BM deciles)

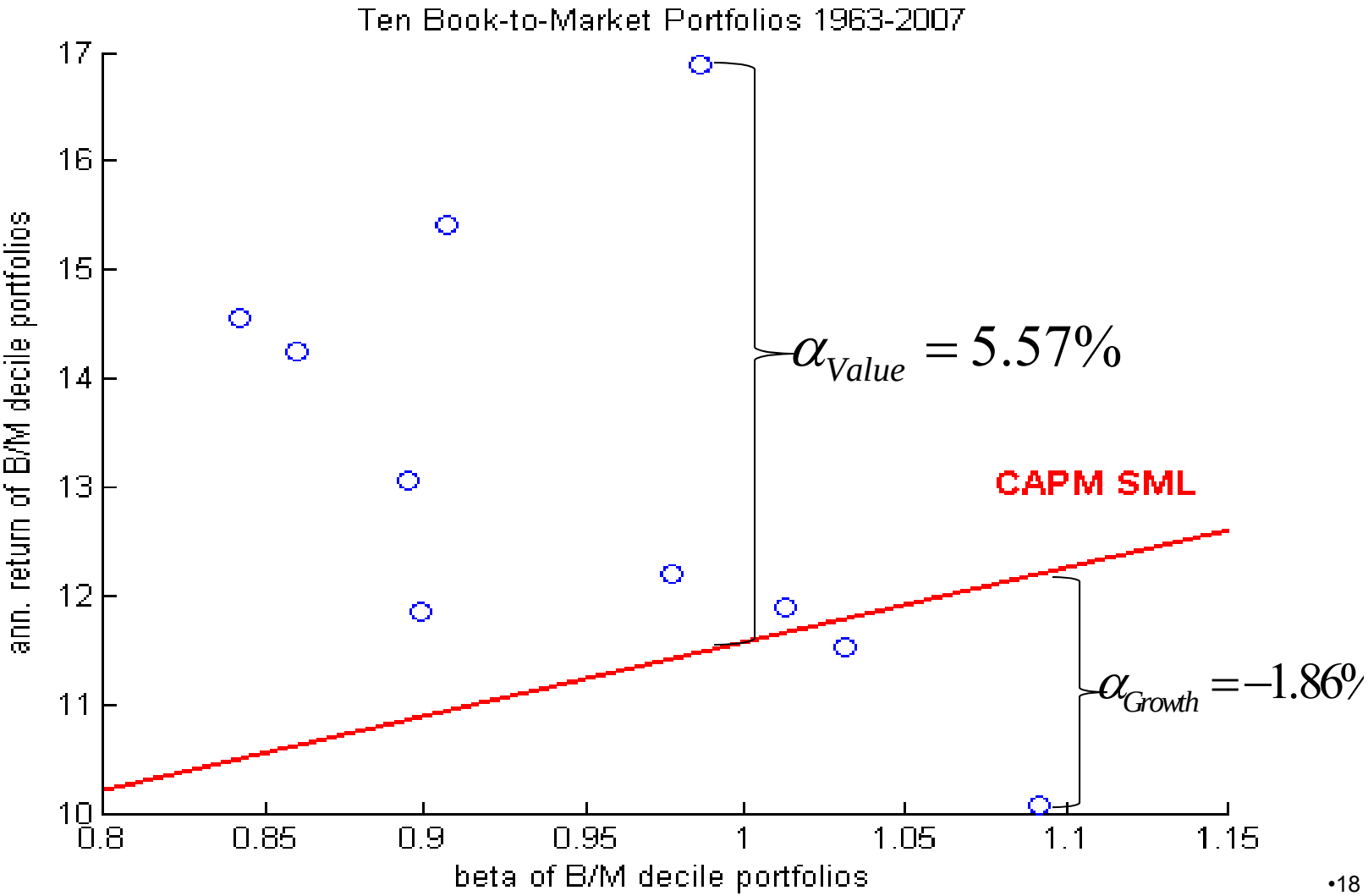
- Average returns 1963-2007 data (540 months):
  - 10^{th} B/M decile (value stocks): avg. annual return = 16.89%,
  - 1^{st} B/M decile (growth stocks): avg. annual return = 10.08%,
  - Value spread = 16.89 - 10.08 =6.81% per year

- CAPM Alpha 1963-2007 (540 months):
  - 10^{th} B/M decile (value stocks): α =5.57%
  - 1^{st} B/M decile (growth stocks): α = -1.86%

The Value Effect



The Value Effect



Fama-French Three-Factor Model

- Returns are explained by size, book to market and by beta.
  - Fama and French (1993)

- Size and book-to-market ratios explain returns on securities.
  - Smaller firms experience higher returns(size style).
  - High book to market firms experience higher returns (value style).

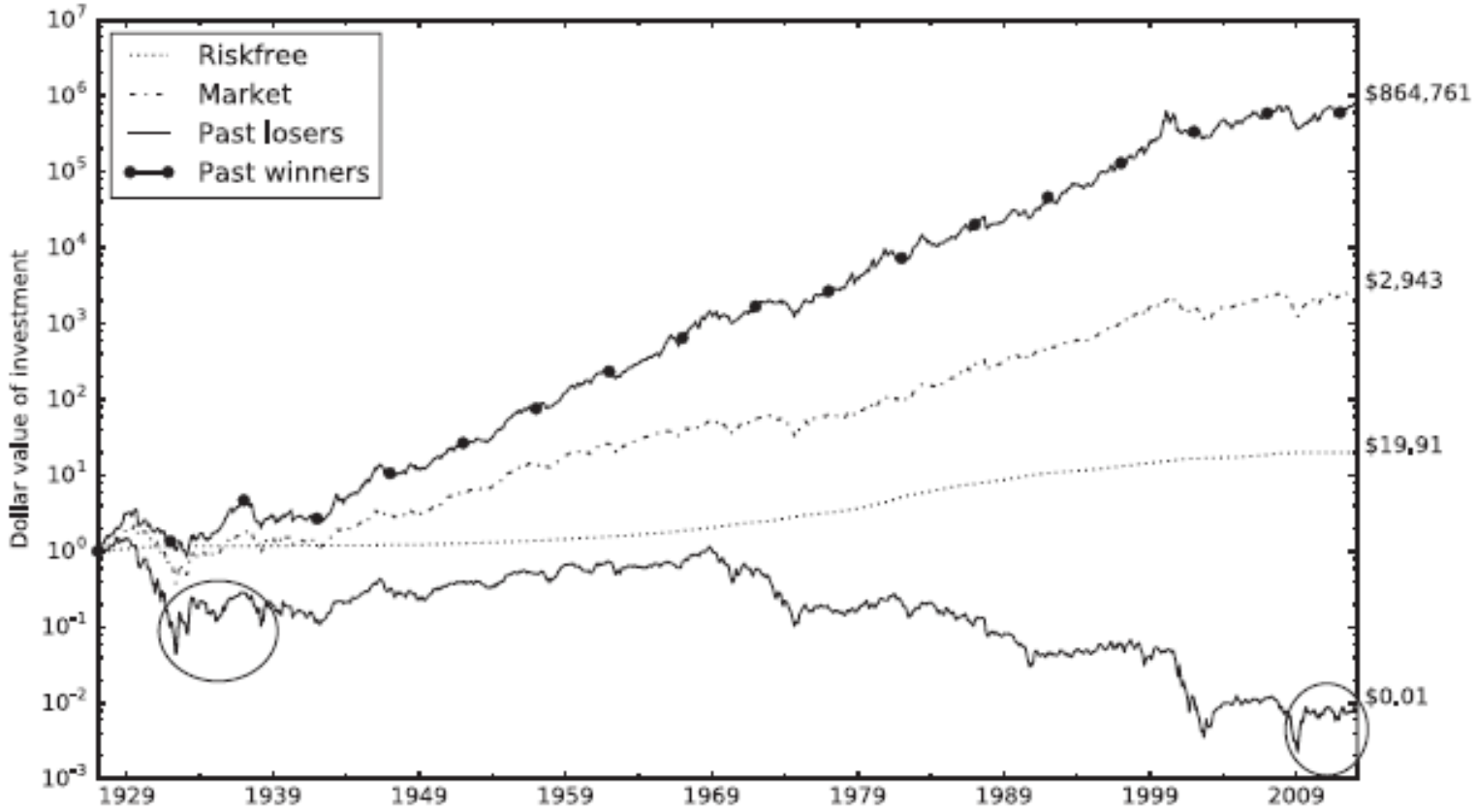
$$r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{Mt} - r_{ft} \right) + s_i SMB_t + h_i HML_t + e_{it}$$

- Zero-investment mimicking portfolios for risk factors
  - Size (SMB)
  - Book-to-market ratio (HML)

https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

The Momentum Effect

- Jegadeesh and Titman (1993) find that past recent winners earned higher average returns than past recent losers.



Daniel, Kent and Tobias Moskowitz (2016) Momentum crashes. Journal of Financial Economics.

The Momentum Effect

- Momentum is employed by most quantitative managers (Swaminathan, 2010).
  - Grinblatt and Titman (1989, 1993), Carhart (1997), and subsequent empirical work suggest that mutual funds also employ momentum.
- Historically, momentum strategies deliver high premia.
  - Daniel and Moskowitz (2013) show that over the post WWII period, through 2008, the long-short equity momentum strategy had an average return of 16.5% per annum, a market beta of -0.125, and an annualized Sharpe-ratio of 0.82.

Carhart Four-Factor Model

- A common extension of the Fama-French three-factor model is the four-factor model of Carhart (1997, JF).

$$r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{Mt} - r_{ft} \right) + s_i SMB_t + h_i HML_t + u_i UMD_t + e_{it}$$

- Adds to excess market return, SMB, and HML, a momentum measure: UMD.
- Usually measured as on the French website:
  - Calculate returns over the eleven months ending one month prior to the observation month, *i.e.,* (*t*-12 to *t*-2).
  - Form equally weighted portfolios of the top 30% (winners) and bottom 30% (losers) by NYSE breakpoints
  - MOM=return to winners-losers in month *t*.

https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_mom_factor.html

The Profitability Effect

- Novy-Marx (2013, Journal of Financial Economics) document that profitability, measured by gross profits-to-assets, has a strong predictive relation with the cross section of stock returns. Profitable firms earn higher returns than unprofitable firms.
- Ball et al. (2015, Journal of Financial Economics) show that profitability, proxied by operating profits scaled by book equity, is positively related to the cross section of stock returns.

The Profitability Effect

Portfolio	r^e	α	MKT	SMB	HML
Panel A: Portfolios sorted on gross profits-to-assets					
Low	0.31	−0.18	0.94	0.04	0.15
	[1.65]	[−2.54]	[57.7]	[1.57]	[5.87]
2	0.41	−0.11	1.03	−0.07	0.20
	[2.08]	[−1.65]	[67.5]	[−3.13]	[8.51]
3	0.52	0.02	1.02	−0.00	0.12
	[2.60]	[0.27]	[69.9]	[−0.21]	[5.42]
4	0.41	0.05	1.01	0.04	−0.24
	[1.94]	[0.83]	[70.6]	[1.90]	[−11.2]
High	0.62	0.34	0.92	−0.04	−0.29
	[3.12]	[5.01]	[58.3]	[−2.03]	[−12.3]
High−low	0.31	0.52	−0.03	−0.08	−0.44
	[2.49]	[4.49]	[−0.99]	[−2.15]	[−10.8]

Novy-Marx, Robert (2013) The other side of value: The gross profitability premium. Journal of Financial Economics.

The Investment Effect

- Titman, Wei, and Xie (2004, Journal of Financial and Quantitative Analysis) show that firms with substantially increase capital investments tend to achieve low benchmark-adjusted returns.
- They argue that this finding is consistent with the hypothesis that investors tend to underreact to the empire building implications of increased investment expenditures.

- Cooper, Gulen, and Schill (2008, Journal of Finance) find that firms with low (high) asset growth rates earn subsequent risk-adjusted returns of 9.1% (-10.4%) per annum.
- Stock prices do quite well in the past years in which capital expenditures increase.

Fama-French Five-Factor Model

- Returns are explained by size, book to market, operating profitability, investment and by beta. -Fama and French (2015)
- Operating profitability and investment factors are added to explain returns on securities.
  - Robust-profitability firms experience higher returns than weak-profitability firms.
  - Low-investment (conservative) firms experience higher returns than high-investment (aggressive) firms.

$$r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{Mt} - r_{ft} \right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

- New zero-investment mimicking portfolios for risk factors:
  - Robust minus weak (RMW)
  - Conservative minus aggressive(CMA)

https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html

Hou, Xue, and Zhang' q-factor model

- Hou, Xue, and Zhang (2014, Review of Financial Studies) suggest that an empirical model that consists of the market, size, investment and profitability factor can largely summarizes cross-sectional stock returns:

$$r_t^i - r_t^f = \alpha^i + \beta_{MKT}^i \left(r_{Mt} - r_{ft} \right) + \beta_{ME}^i r_{ME,t} + \beta_{I/A}^i r_{I/A,t} + \beta_{ROE}^i r_{ROE,t} + e_t^i$$

- Take the intersections of the 2 size, 3 I/A, and 3 ROE groups to form 18 portfolios.
  - Size: Small-minus-big, i.e., 9 small size portfolios minus 9 big size portfolios
  - I/A: Low-minus-high, i.e., 6 low I/A portfolios minus 6 high I/A portfolios
  - ROE: High-minus-low, i.e., 6 high ROE portfolios minus 6 low ROE portfolios

Lecture Summary

- Many studies provide evidence on the return premia to value and momentum strategies globally across asset classes.

- Both Fama-French three-factor and Carhart four-factor models are very often used by practitioners, such as portfolio managers, regulatory commissions and companies(capital budgeting decisions).

- However, value and momentum pose a challenge to risk-based and behavioural-based theories to accommodate.

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Required readings:

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